Semileptonic decays $B \to (\pi, \rho)e\nu$ in relativistic quark model

D. Melikhov

Nuclear Physics Institute, Moscow State University, Moscow, 119899, Russia
Electronic address: melikhov@monet.npi.msu.su

Quark model results for the $B \to \pi, \rho$ decays are analysed, making use of the dispersion formulation of the model: The form factors at $q^2 > 0$ are expressed as relativistic invariant double spectral representation over invariant masses of the initial and final mesons through their light–cone wave functions. The dependence of the results on the quark model parameters is studied. For various versions of the quark model the ranges $\Gamma(\bar{B}^0 \to \pi^+ e\bar{\nu}) = (7{\pm}2){\times}10^{12}|V_{ub}|^2\ s^{-1}$, $\Gamma(\bar{B}^0 \to \rho^+ e\bar{\nu})/\Gamma(\bar{B}^0 \to \pi^+ e\bar{\nu}) = 1.45{\pm}0.1$, and $\Gamma_L/\Gamma_T = 0.7 \pm 0.08$ are found. The effects of the constituent quark transition form factor are briefly discussed.
PACS number(s): 13.20.-v, 12.39.Hg, 12.39.Ki.

Weak decays of hadrons provide an important source of information on the parameters of the standard model of electroweak interactions, the structure of weak currents, and internal structure of hadrons. Hadron decay rates involve both the Cabibbo–Kobayashi–Maskawa matrix elements and hadron form factors, therefore the extraction of the standard model parameters from the experiments on hadron decays requires reliable information on hadron structure.

A theoretical study of hadronic matrix elements of the weak currents inevitably encounters the problem of describing the hadron structure and requires a nonperturbative consideration. This gives the main uncertainty to the theoretical predictions for hadron transition amplitudes.

For the transitions between hadrons each containing a heavy quark and light degreess of freedom, the number of the independent form factors is considerably reduced due to the heavy quark symmetry [1]. For instance, in the leading $1/m_Q$ order the transition between heavy mesons is described in terms of the single Isgur–Wise function, which should be estimated within a nonperturbative approach. The $O(1/m_Q^N)$ corrections to this picture can be consistently calculated within the Heavy Quark Effective Theory [2], an effective theory based on QCD in the limit of large quark masses (a detailed review can be found in [3]).

For the decays caused by the heavy–to–light quark transitions the argumentation of the heavy–to–heavy transitions does not work, and the situation turns out to be much less definite. For instance, for the decays $B \to \pi, \rho$ the uncertainty of the results of nonperturbative approaches such as the quark model [4]–[9], QCD sum rules [10]–[22], and lattice calculations [13]–[15] is too large to draw any definite conclusion on the form factor values and decay rates (see Tables 1, 2).

A step forward in the understanding of heavy–to–light transitions was recently done by B.Stech who noticed that relations between heavy–to–light form factors can be obtained if use is made of the constituent quark picture [16]. Namely, assuming that (i) the momentum distribution of the constituent quarks inside a meson is strongly peaked with a width corresponding to the confinement scale, and (ii) the process in which the spectator retains its spin and momentum dominates the transition amplitude, [1] the 6 form factors can be reduced to a single function just as it is in the case of the heavy–to–heavy transition. These relations are expected to be valid up to the corrections $O\left(2m_u M_B/(M_B^2 + M_\pi^2 - q^2)\right)$. Although these corrections cannot be estimated numerically, the relations can be a guideline to the analysis of the heavy–to–light decay processes.

Constituent quark picture has been extensively applied to the description of the decay processes [4]–[9], [17]. Although in the first models by Wirbel, Stech, and Bauer (WSB) [4] and Isgur, Scora,

[1] Actually, one more assumption on the dynamics of the procees is employed. Namely, the picture in [16] includes both the constituent and current quarks. And for deriving the final relations it is important that the momentum of the current quark coincides with the momentum of the corresponding constituent quark. This assumption allows one to avoid the appearence of the constituent quark transition form factor which should be taken into account if the picture with only constituent quarks is considered.

Grinstein and Wise (ISGW) [6] quark spins were not treated relativistically, it has become clear soon that for a consistent application of quark models to electroweak decays, a relativistic treatment of quark spins is necessary [3]. The exact solution to this complicated dynamical problem is not known, but a simplified self–consistent relativistic treatment of the quark spins can be performed within the light–cone formalism [18]. The only difficulty with this approach is that the applicability of the model is restricted by the condition $q^2 \le 0$, while the physical region for hadron decays is $0 \le q^2 \le (M_i - M_f)^2$, $M_{i,f}$ being the initial and final hadron mass, respectively. So, for obtaining the form factors in the physical region and decay rates and lepton distributions, assumptions on the form factor q^2–behavior were necessary. A procedure to remedy this difficulty has been proposed in [19].

The approach of [19] is based on the dispersion formulation of the light–cone quark model [20]. Namely, the transition form factors obtained within the light–cone formalism at $q^2 < 0$ [17], are represented as dispersion integrals over initial and final hadron masses. The transition form factors at $q^2 > 0$ are derived by performing the analytic continuation in q^2 from the region $q^2 \le 0$. As a result, for a decay caused by the weak transition of the quark $Q(m_i) \to Q(m_f)$, form factors in the region $q^2 \le (m_i - m_f)^2$ are expressed through the light–cone wave functions of the initial and final hadrons. We apply this approach to the analysis of the $B \to (\pi, \rho)$ decays and study the dependence of the results on the quark model parameters, such as constituent quark masses and wave functions. We also check the fulfillment of the Stech relations in particular model calculations.

The amplitudes of the semileptonic decays of a pseudoscalar meson $P(M_1)$ into the final pseudoscalar $P(M_2)$ and vector $V(M_2)$ mesons have the following structure [1]

$$\begin{aligned}
<P(M_2,p_2)|V_\mu(0)|P(M_1,p_1)> &= f_+(q^2)(p_1+p_2)_\mu + f_-(q^2)(p_1-p_2)_\mu \\
<V(M_2,p_2,\epsilon)|V_\mu(0)|P(M_1,p_1)> &= 2g(q^2)\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}(p_2)p_1^\alpha p_2^\beta \qquad (1)\\
<V(M_2,p_2,\epsilon)|A_\mu(0)|P(M_1,p_1)> &= if(q^2)\epsilon^*_\mu + ia_+(q^2)(\epsilon^* p_1)(p_1+p_2)_\mu + ia_-(q^2)(\epsilon^* p_1)(p_1-p_2)_\mu
\end{aligned}$$

We denote both the pseudoscalar and vector meson masses as M_2 but the relevant value is taken in each case.

Our goal is to calculate the form factors of the transition between S-wave mesons at $0 \le q^2 \le (M_1 - M_2)^2$ within the constituent quark picture. In this picture the initial meson $P(M_1)$ is a bound state of the constituent quarks $Q(m_2)\bar{Q}(m_3)$, the final meson is an S-wave bound state $Q(m_1)\bar{Q}(m_3)$, and the transition process is described by the graph of Fig.1.

We however start with the region $q^2 < 0$ and make use of the fact that the transition form factors calculated within the light–cone quark model [17] can be written as double spectral representations [19],[20] over the invariant masses of the initial and final mesons

$$f_i(q^2) = f_{21}(q^2) \int\limits_{(m_1+m_3)^2}^{\infty} \frac{ds_2 G_2(s_2)}{\pi(s_2 - M_2^2)} \int\limits_{s_1^-(s_2,q^2)}^{s_1^+(s_2,q^2)} \frac{ds_1 G_1(s_1)}{\pi(s_1 - M_1^2)} \frac{\tilde{f}_i(s_1,s_2,q^2)}{16\lambda^{1/2}(s_1,s_2,q^2)}, \qquad (2)$$

where

$$s_1^\pm(s_2,q^2) = -\frac{1}{2m_1^2}$$

$$\times \Big(s_2(q^2 - m_1^2 - m_2^2) - q^2(m_1^2 + m_3^2) + (m_1^2 - m_2^2)(m_1^2 - m_3^2) \pm \lambda^{1/2}(s_2, m_3^2, m_1^2)\lambda^{1/2}(q^2, m_1^2, m_2^2) \Big)$$

and

$$\lambda(s_1,s_2,s_3) = (s_1 + s_2 - s_3)^2 - 4s_1 s_2.$$

Here $G_{1,2}$ are the vertex functions which describe the constituent quark structure of the mesons, $f_{21}(q^2)$ is the form factor of the constituent quark weak transition $m_2 \to m_1$. In what follows we adopt the conventional approximation $f_{21}(q^2) = 1$. The same representations of the form factors has been obtained in [21] taking into account the contribution of two–particle singularities in the Feynman graphs.

The double spectral densities $\tilde{f}_i$ of the form factors f_+, g, a_+, and f, which give a nonvanishing contribution to the cross–section of the semileptonic decay in the case of zero lepton mass, read (hereafter $s_3 \equiv q^2$)

$$\tilde{f}_+(s_1,s_2,s_3) = D + (\alpha_1+\alpha_2)D_3, \tag{3}$$

$$\tilde{g}(s_1,s_2,s_3) = 2\left[m_1\alpha_2 + m_2\alpha_1 + m_3(1-\alpha_1-\alpha_2) - \frac{2\beta}{\sqrt{s_2}+m_1+m_3}\right], \tag{4}$$

$$\begin{aligned}\tilde{a}_+(s_1,s_2,s_3) = {} & 2\left[2m_2\alpha_{11} + 2m_3(\alpha_1-\alpha_{11}) - m_1\alpha_2 - m_2(\alpha_1-2\alpha_{12})\right. \\ & \left. -m_3(1-\alpha_1-\alpha_2-2\alpha_{12}) + \frac{C\alpha_1 + C_3(\alpha_{11}+\alpha_{12})}{\sqrt{s_2}+m_1+m_3}\right],\end{aligned} \tag{5}$$

$$\tilde{f}(s_1,s_2,s_3) = \frac{M_2}{\sqrt{s_2}}\tilde{f}_D(s_1,s_2,s_3) + M_2\tilde{a}_+(s_1,s_2,s_3)\left(\frac{s_1-s_2-s_3}{2\sqrt{s_2}} - \frac{M_1^2-M_2^2-s_3}{2M_2}\right), \tag{6}$$

$$\tilde{f}_D(s_1,s_2,s_3) = 4\left[E + 2\beta(m_2-m_3) + \frac{C_3\beta}{\sqrt{s_2}+m_1+m_3}\right]$$

where

$$D = s_1 + s_2 - (m_2-m_3)^2 - (m_1-m_3)^2, \quad D_3 = s_3 - (m_1-m_2)^2 - D,$$
$$C = s_1 + s_2 - (m_2-m_3)^2 - (m_1+m_3)^2, \quad C_3 = s_3 - (m_1+m_2)^2 - C,$$
$$E = m_1m_2m_3 + \frac{m_2}{2}(s_2-m_1^2-m_3^2) + \frac{m_1}{2}(s_1-m_2^2-m_3^2) - \frac{m_3}{2}(s_3-m_1^2-m_2^2),$$
$$\alpha_1 = \frac{1}{\lambda(s_1,s_2,s_3)}\left[(s_1+s_2-s_3)(s_2-m_1^2+m_3^2) - 2s_2(s_1-m_2^2+m_3^2)\right],$$
$$\alpha_2 = \frac{1}{\lambda(s_1,s_2,s_3)}\left[(s_1+s_2-s_3)(s_1-m_2^2+m_3^2) - 2s_1(s_2-m_1^2+m_3^2)\right],$$
$$\beta = \frac{1}{4}\left[2m_3^2 - \alpha_1(s_1-m_2^2+m_3^2) - \alpha_2(s_2-m_1^2+m_3^2)\right],$$
$$\alpha_{11} = \alpha_1^2 + 4\beta\frac{s_2}{\lambda(s_1,s_2,s_3)}, \quad \alpha_{12} = \alpha_1\alpha_2 - 2\beta\frac{s_1+s_2-s_3}{\lambda(s_1,s_2,s_3)}.$$

For an S-wave meson (vector and pseudoscalar) with the mass M built up of the constituent quarks m_1 and m_2, the function G is normalized as follows [19]

$$\int \frac{G^2(s)ds}{\pi(s-M^2)^2}\frac{\lambda^{1/2}(s,m_1^2,m_2^2)}{8\pi s}(s-(m_1-m_2)^2) = 1. \tag{7}$$

Notice that the double dispersion representation without subtractions are valid for the form factors f_+, f_- g, a_+, and a_-, while the form factor f requires subtractions.

Now, we obtain the form factors at $q^2>0$ by performing the analytic continuation in q^2. For the function which has at $q^2 \equiv s_3 \le 0$ the structure

$$\phi(s_3) = \int\limits_{(m_1+m_3)^2}^{\infty}\frac{ds_2G_2(s_2)}{\pi(s_2-M_2^2)}\int\limits_{s_1^-}^{s_1^+}\frac{ds_1G_1(s_1)}{\pi(s_1-M_1^2)}\left[\frac{P_0(s_1,s_2,s_3)}{\lambda^{1/2}(s_1,s_2,s_3)} + \frac{P_1(s_1,s_2,s_3)}{\lambda^{3/2}(s_1,s_2,s_3)} + \frac{P_2(s_1,s_2,s_3)}{\lambda^{5/2}(s_1,s_2,s_3)}\right], \tag{8}$$

where P_i are polynomials of s, the analytical continuation to the region $q^2>0$ yields the following expression at $q^2 \le (m_2-m_1)^2$

$$\phi(s_3) = \int\limits_{(m_1+m_3)^2}^{\infty}\frac{ds_2G_2(s_2)}{\pi(s_2-M_2^2)}\int\limits_{s_1^-}^{s_1^+}\frac{ds_1G_1(s_1)}{\pi(s_1-M_1^2)}\left[\frac{P_0(s_1,s_2,s_3)}{\lambda^{1/2}(s_1,s_2,s_3)} + \frac{P_1(s_1,s_2,s_3)}{\lambda^{3/2}(s_1,s_2,s_3)} + \frac{P_2(s_1,s_2,s_3)}{\lambda^{5/2}(s_1,s_2,s_3)}\right] \tag{9}$$

$$
+2\theta(s_3)\int\limits_{s_2^0}^{\infty}\frac{ds_2 G_2(s_2)}{\pi(s_2-M_2^2)}\int\limits_{s_1^R}^{\infty}\frac{ds_1}{\pi(s_1-s_1^R)^{1/2}}\left[\tilde\phi_0(s_1)+\frac{\tilde\phi_1(s_1)-\tilde\phi_1(s_1^R)}{s_1-s_1^R}+\frac{\tilde\phi_2(s_1)-\tilde\phi_2(s_1^R)-\tilde\phi_2'(s_1^R)(s_1-s_1^R)}{(s_1-s_1^R)^2}\right],
$$

$$
\sqrt{s_2^0}=-\frac{s_3+m_1^2-m_2^2}{2\sqrt{s_3}}+\sqrt{\left(\frac{s_3+m_1^2-m_2^2}{2\sqrt{s_3}}\right)^2+(m_3^2-m_1^2)},\quad s_3<(m_2-m_1)^2, \tag{10}
$$

$s_1^L=(\sqrt{s_2}-\sqrt{s_3})^2$, $s_1^R=(\sqrt{s_2}+\sqrt{s_3})^2$, $\lambda(s_1,s_2,s_3)=(s_1-s_1^R)(s_1-s_1^L)$, and

$$
\phi_n(s)=\frac{G_1(s)P_n(s_1,s_2,s_3)\theta(s_1<s_1^-)}{(s_1-M_1^2)(s_1-s_1^L)^{n+1/2}}. \tag{11}
$$

For deriving this expression it is important that the functions $G_{1,2}$ have no singularities in the r.h.s. of the complex $s-$plane [21]. Along with the normal Landau–type contribution connected with the subprocess when all intermediate particles go on mass shell (the first term in (9)), the anomalous contribution (the second term in (9)) emerges at $q^2>0$ [22]. The normal contribution dominates the form factor at small positive q^2 and vanishes at the 'quark zero recoil point' $q^2=(m_2-m_1)^2$. The anomalous contribution is negligible at small q^2 and grows as $q^2\to(m_2-m_1)^2$.

We are now in a position to apply these results to the $B\to\pi,\rho$ decays, in which case $m_2=m_b$, $m_1=m_3=m_u$. To this end the quark model parameters (m_u, m_b and the vertex functions) should be specified.

For a pseudoscalar meson built up of quarks with the masses m_1 and m_2, it is convenient to introduce the function w related to the vertex function G as follows

$$
G(s)=\frac{\pi}{\sqrt{2}}\frac{\sqrt{s^2-(m_1^2-m_2^2)^2}}{\sqrt{s-(m_1-m_2)^2}}\frac{s-M^2}{s^{3/4}}w(k),\qquad k=\frac{\lambda^{1/2}(s,m_1^2,m_2^2)}{2\sqrt{s}} \tag{12}
$$

The normalization condition (7) for G yields the following normalization condition for w

$$
\int w^2(k)k^2dk=1. \tag{13}
$$

The function w is the ground–state S–wave radial wave function of a pseudoscalar and vector meson for which a simple exponential form is usually chosen

$$
w(k)=\exp(-k^2/2\beta^2) \tag{14}
$$

We consider several sets of the quark model parameters used for the description of the meson spectra and elastic form factors (Table 3).

For analysing the results of calculations, introduce the functions $R_i(q^2)$ such that

$$
\begin{aligned}
f_+(q^2) &= R_+(q^2),\\
V(q^2) &= (M_1+M_2)g(q^2)=(1+r)R_V(q^2),\\
A_1(q^2) &= \frac{1}{(M_1+M_2)}f(q^2)=\frac{1+r^2-y}{1+r}R_1(q^2),\\
A_2(q^2) &= -(M_1+M_2)a_+(q^2)=(1+r)\frac{1-r^2-y}{(1+r)^2-y}R_2(q^2),
\end{aligned}
$$

where $r=M_2/M_1$, $y=q^2/M_1^2$. As found by Stech [16], $R_i(q^2)$ should be equal up to the corrections of the order $O\left(2m_3M_1/(M_1^2+M_2^2-q^2)\right)$. For the decay $B\to\pi$ the corrections about 10% are expected even at $q^2=0$.

Table 4 presents the parameters of the best fits to the calculated form factors in the form

$$
R_i=\frac{R_i(0)}{(1-q^2/M_i^2)^{n_i}}.
$$

These fits approximate the form factors with better than 0.5% accuracy in the relevant kinematic range and can be used for the calculation of the decay rates. In all the form factors, the anomalous contribution comes into the game only at $q^2 \geq 15$ Gev^2; below this point it is negligible. The decay rates are calculated with the form factors from Table 4 via the formulas from Ref.[7]. Table 5 summarises the results on the form factors and decay rates.

The following conclusions can be drawn:

(i) Relativistic effects are important in the $B \to \pi, \rho$ decays, as our results considerably differ from those of the models with nonrelativistic treating quark spins with the same parameters (Sets 1 and 2).

(ii) The functions R_i are equal with an expected 10-20% accuracy as found by Stech for any set of the parameters. However, the magnitude and q^2-dependence of the functions R strongly depend on these parameters (see Fig.2). The 'quark model average values' for the decay rates can be determined as

$$\begin{aligned} \Gamma(\bar{B}^0 \to \pi^+ e\bar{\nu}) &= (7 \pm 2) \times 10^{12} |V_{ub}|^2 \; s^{-1}, \\ \Gamma(\bar{B}^0 \to \rho^+ e\bar{\nu})/\Gamma(\bar{B}^0 \to \pi^+ e\bar{\nu}) &= 1.45 \pm 0.1, \\ \Gamma_L/\Gamma_T &= 0.7 \pm 0.08. \end{aligned} \tag{15}$$

These values are not far from the predictions of other models (Table 2), except for the results by Narison ([12]). This is mainly due to a very specific decreasing q^2–behavior of A_1 in [12]. [2]

The ratio of the decay rates have rather good accuracy, while their absolute values are less definite. Perhaps, the way out of this situation lies in a more detailed consideration of the constituent quark transition form factor. We use an approximation $f_{21} \equiv 1$. However, there are arguments in favour of a nontrivial q^2–dependence of f_{21}. Actually, within the constituent quark picture, the constituent quark form factor arises from the two sources. First, it appears as a bare form factor which describes the constituent quark amplitude of the weak current defined through current quarks. One can assume this bare form factor to be close to unity at relevant q^2. Secondly, this bare form factor is renormalised by the constituent quark rescatterings (final state interactions) [20]. These very interactions yield meson formation and are not small at least in the region of q^2 near the meson mass. So the relation $f_{21} = const$ cannot be valid for all q^2. Whether it is strongly violated or not at the q^2 of interest is not clear yet. Setting $f_{21} = 1$ in (9) means that we take into account the effects of the constituent quark binding in the initial and final meson channels, and neglect the quark rescatterings in the q^2–channel. The constituent quark form factor should depend on the parameters of the model. One can expect that once the constituent quark form factor is taken into account thoroughly, the results on the meson transition form factors and decay rates will be weakly dependent on the quark model parameters. In the ratio of the decay rates the constituent quark form factors cancel, and these results seem to be more reliable.

I am grateful to V. Anisovich, S. Godfrey, I. Grach, I. Narodetskii, and K. Ter–Martirosyan for valuable discussions and to V. Nikonov for help in numerical calculations. The work was supported by the Russian Foundation for Basic Research under grant 95–02–04808a.

[2] The decay rates are rather sensitive to the details of the q^2–behaviour of A_1. In the light–cone approach the form factor A_1 has a specific feature: unlike other form factors it cannot be determined uniquely. Using the dispersion language, the form factor A_1 requires subtractions which cannot be fixed uniquely. This is a general situation for any amplitude with a vector particle in the initial or final state. Such an amplitude is transverse with respect to the momentum of the vector particle, and this yields a necessity of subtractions in some of the form factors. However, different definitions of the subtraction procedure lead to rather close results.

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Table 1: The form factors of the decays $B \to \pi, \rho$ at $q^2 = 0$. The labels QM, SR, and LAT stand for Quark Model, Sum Rules, and Lattice, respectively.

Ref.		$f_+(0)$	$V(0)$	$A_1(0)$	$A_2(0)$	$V(0)/A_1(0)$	$A_2(0)/A_1(0)$
QM	WSB [4]	0.33	0.33	0.28	0.28	1.2	1.0
	ISGW [6]	0.09	0.27	0.05	0.02	5.4	0.4
SR	BBD [10]	0.24± 0.025					
	Ball [11]	0.26± 0.02	0.6±0.2	0.5±0.1	0.4±0.2		
	Narison [12]	0.23± 0.02	0.45±0.05	0.38±0.04	0.45±0.05	1.11± 0.01	1.11± 0.01
Lat	APE [13]a	0.29± 0.06	0.45±0.22	0.29±0.16	0.24± 0.56	2.0±0.9	0.8±1.5
	APE [13]b	0.35± 0.08	0.53±0.31	0.24±0.12	0.27± 0.80	2.6±1.9	1.0±3.1
	ELC [14]a	0.26± 0.16	0.34±0.10	0.25±0.06	0.38± 0.22	1.4±0.2	1.5±0.7
	ELC [14]b	0.30± 0.19	0.37±0.11	0.22±0.05	0.49± 0.26	1.6±0.3	2.3±0.9

Table 2: Decay rates of the transitions $\bar{B}^0 \to (\pi^+, \rho^+) e^- \bar{\nu}$ in units $|V_{ub}|^2 \times 10^{12}\ s^{-1}$.

Ref.	$\Gamma(B \to \pi)$	$\Gamma(B \to \rho)$	Γ_ρ/Γ_π	Γ_L/Γ_T
WSB [4]	7.43	26.1	3.5	1.34
ISGW [6]	2.1	8.3	3.95	0.75
Ball [11]	5.1± 1.1	12±4		0.06±0.02
Narison [12]	3.6± 0.6		0.9±0.2	0.2±0.01
APE [13]	8±4			

Table 3: Parameters of the quark model.

Parameter Set	m_u	m_b	β_π	β_B
Set 1 [4]	0.35	4.9	0.4	0.4
Set 2 [6]	0.33	5.12	0.31	0.41
Set 3 [17]	0.29	4.9	0.29	0.386
Set 4 [23]	0.25	4.7	0.32	0.55

Table 4: Parameters of the best fits to the calculated form factors.

Set	$R_+(0)$	M_+	n_+	$R_V(0)$	M_V	n_V	$R_1(0)$	M_1	n_1	$R_2(0)$	M_2	n_2
Set 1	0.29	6.29	2.35	0.30	6.28	2.36	0.27	7.07	2.65	0.25	6.13	2.17
Set 2	0.20	6.22	2.45	0.20	6.22	2.46	0.20	6.78	2.65	0.19	6.00	2.34
Set 3	0.21	5.90	2.33	0.21	5.90	2.35	0.21	6.50	2.70	0.20	5.90	2.45
Set 4	0.26	5.44	1.72	0.29	5.46	1.73	0.29	5.68	1.67	0.28	5.36	1.67

Table 5: Form factors of the $\bar{B}^0 \to (\pi^+, \rho^+) e^- \bar{\nu}$ decays at $q^2 = 0$ and the calculated decay rates in units $|V_{ub}|^2 \times 10^{12}\ s^{-1}$.

Set	$f_+(0)$	$V(0)$	$A_1(0)$	$A_2(0)$	$\Gamma(B \to \pi)$	$\Gamma(B \to \rho)$	Γ_ρ/Γ_π	Γ_L/Γ_T
Set 1	0.29	0.34	0.24	0.21	9.2	12.6	1.38	0.77
Set 2	0.20	0.22	0.17	0.16	5.0	7.25	1.45	0.73
Set 3	0.20	0.24	0.19	0.17	6.1	8.8	1.44	0.62
Set 4	0.26	0.33	0.26	0.24	8.9	13.8	1.55	0.67

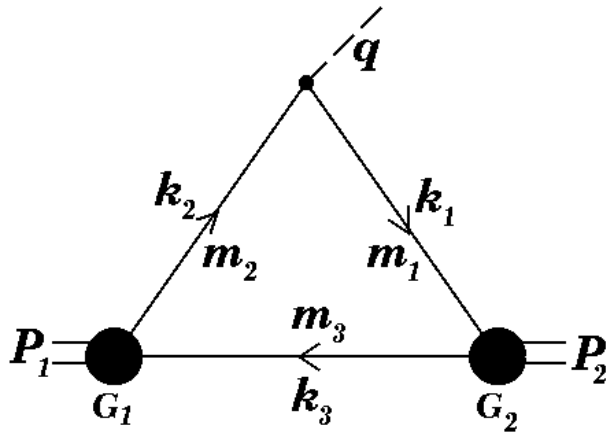

Figure 1: One-loop graph for a meson decay.

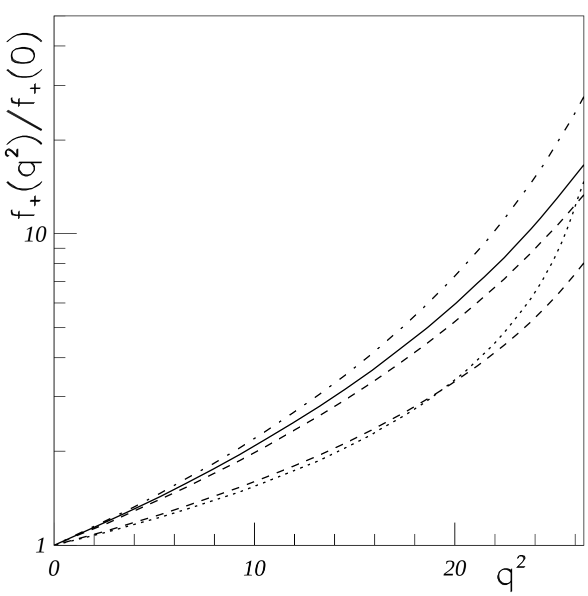

Figure 2: The q^2–dependence of the form factor f_+ for the decay $b \to \pi$ at various quark model parameters. Set 1 – upper dashed line, set 2 – lower solid line, set 3 – dash–dotted line, set 4 – lower dashed line, the monopole formula with $M_{pole} = M_{B^*} = 5.34\,\mathrm{GeV}$ – dotted line.